Latent State-Trait Theory

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1 Introduction

2 Latent Variables in Classical Test Theory

2.1 The Kind of Random Experiments Considered

In classical test theory, we consider the following kind of empirical phenomenon:

- (a) Sample a person u from a set Ω_U of persons, and
- (b) observe the behavior *o* of the person *u*, i.e., observe an element *o* of the set Ω_O of possible observations.

The set Ω_O of possible observations is itself a set product $\Omega_O = \Omega_{O_1} \times \ldots \times \Omega_{O_m}$ of m sets of possible observations. If we just consider two real-valued measurements (e.g., of cortisol in saliva, two scores indicating depression, two scores indicating anxiety, two scores indicating neuroticism, etc.), then $\Omega_O = \Omega_{O_1} \times \Omega_{O_2} = \mathbb{R} \times \mathbb{R}$, and the observation o is a pair of two real numbers, i.e., o = (a, b), where $a, b \in \mathbb{R}$.

2.2 The Mathematical Representation of the Random Experiment

The kind of empirical phenomenon described by (a) and (b) is called a *random experiment*. The set of possible outcomes of such a random experiment is Ω [see Eq. (2) in Table 1]. Its elements are all possible outcomes $\omega = (u, o) = (u, o_1, ..., o_m)$. On this set Ω we choose an appropriate set of events, a σ -*algebra* \mathcal{A} of subsets of Ω , and assume that there is a (usually unknown) *probability measure* P on \mathcal{A} . The triple (Ω, \mathcal{A}, P) , which is called a *probability space*, is the mathematical representation — and describes the mathematical structure including the probabilities of all events — of the random experiment considered.

2.3 The Random Variables

A *random variable* on (Ω, \mathscr{A}, P) , say Y, is a mapping on the domain Ω with values in a set, say Ω' , which is denoted by $Y: \Omega \to \Omega'$. The person variable $U: \Omega \to \Omega_U$ is a projection mapping that maps each possible outcome $\omega \in \Omega$ into the set Ω_U . That is, if the random experiment is actually conducted and the person u is sampled, then a value $U(\omega)$ of the person variable U is u. In contrast, the test score variables Y_1, \ldots, Y_m map each possible outcome $\omega \in \Omega$ into the set \mathbb{R} of real numbers. We assume that these random variables are nonnegative or have finite expectations. Under this assumption, the U-conditional expectation $E(Y_i | U)$ — synonymously called the regression of Y_i on U — exists for each $i = 1, \ldots m$. These regressions are random variables on (Ω, \mathscr{A}, P) too, and their values are

$$E(Y_i | U)(\omega) = E(Y_i | U = u), \quad \text{if } U(\omega) = u. \tag{1}$$

In other words, the values of a *U*-conditional expectation $E(Y_i | U)$ are the (U=u)-conditional expectations $E(Y_i | U=u)$.

So far, we only described the random experiment considered and its mathematical representation (Ω, \mathcal{A}, P) , the probability space, as well as some random variables on this space. In a sense, the probability space (Ω, \mathcal{A}, P) and the random variables U, Y_i , and $E(Y_i | U)$, i = 1, ..., m, mentioned above are the primitives, on which the basic concepts of classical test theory can be introduced.

Table 1. Basic concepts of CTT			
Primitives			
The set of possible outcomes of the random experiment	$\Omega = \Omega_U \times \Omega_O$	(2)	
Manifest random variables	$Y_i\colon\Omega\to\mathbb{R}$	(3)	
Person variable	$U\colon\Omega\to\Omega_U$	(4)	
Theoretical variables of CTT			
True score variables	$\tau_i := E(Y_i U)$	(5)	
Measurement error variables	$\varepsilon_i := Y_i - \tau_i$	(6)	
Note ???			

2.4 True Score and Measurement Error Variables

The two basic concepts of CTT, *true-score* and *measurement error variables* are defined by Equations (5) and (6) (see Table 1). In Table 2 we present some properties that are implied by the definitions of true-score and measurement error variables in Equations (5) and (6). These properties are always true provided that the expectations $E(Y_i)$ and the variances $Var(Y_i)$, i = 1, ..., m, are finite.

In Figure 1 some of these properties are represented in a path diagram. The most important ones are:

- (a) Each manifest variable Y_i has its own true-score variable τ_i and its own measurement error variable ε_i .
- (b) True-score variables may correlate with each other.
- (c) Measurement error variables may correlate with each other.
- (d) True-score variables on one side and measurement error variables on the other side are uncorrelated.

To emphasize, if we assume that the expectations and variances of the Y_i are finite, then these properties cannot be wrong in any empirical application. They are not assumptions. Instead, they are logical implications of the definitions of true-score and measurement error variables. They are always true, just in the same way as it is always true that a bachelor is unmarried. No empirical study can falsify this fact; it may only reveal that some alleged bachelors are actually no bachelors. Similarly, no empirical study can falsify that true-score and measurement error variables are uncorrelated. It may only show that an alleged pair of true-score and measurement error variables.

2.5 Models of CTT

Introducing models of CTT, we assume that the true-score variables τ_i are 'identical' in one of the meanings specified by Equations (17) to (19) in Table 3. In all three models, the *common latent variable*, τ , is defined by $\tau := \tau_1$. Of course, in the models of τ -equivalent and τ -congeneric variables, this definition is arbitrary to some degree. For example, in the model of τ -equivalent variables, defining $\tau^* := \alpha + \tau_1$,

Table 2. Properties of true-score and measurement error variables

$$Y_i = \tau_i + \varepsilon_i \tag{7}$$

 $Var(Y_i) = Var(\tau_i) + Var(\varepsilon_i)$ (8) $Cov(\tau_i,\varepsilon_i) = 0$

$$\begin{aligned} (i_i, \varepsilon_j) &= 0 \end{aligned}$$

$$E(\varepsilon_i) = 0 \tag{10}$$
$$E(\varepsilon_i | U) = 0 \tag{11}$$

$$E[\varepsilon_i | f(U)] = 0, \text{ for all mappings } f(U) \text{ of } U.$$
(12)

Note. All properties in this table are implied by the definition of true-score and measurement error variables. They are always true if the expectations $E(Y_i)$ and the variances $Var(Y_i)$ exist.

 $\alpha \in \mathbb{R}$, would do as well. Hence, under essential τ -equivalence, τ is *uniquely defined up to translations.* In this case we say that τ has a *difference scale*. Similarly, under τ -congenericity, choosing any linear function $\tau^* := \alpha + \beta \cdot \tau_1$, $\alpha, \beta \in \mathbb{R}$, $\beta > 0$, would do as well. This shows that, under τ -congenericity, τ is uniquely defined up to positive *linear transformations.* In this case we say that τ has an *interval scale*. Note that in all three models, the common latent variable τ is a (deterministic) function of each of the true-score variables. Therefore, it is also a function of the person variable U, and this means that each of its values characterize a person.

Using $\varepsilon_i = Y_i - \tau_i$, Equation (17) implies

$$Y_i = \tau_i + \varepsilon_i = \tau + \varepsilon_i, \quad \text{for all } i = 1, \dots, m, \tag{13}$$

Equation (18) implies

$$Y_i = \tau_i + \varepsilon_i = \lambda_{i0} + \tau + \varepsilon_i, \quad \text{for all } i = 1, \dots, m, \tag{14}$$

and Equation (19) implies

$$Y_i = \tau_i + \varepsilon_i = \lambda_{i0} + \lambda_{i1} \cdot \tau + \varepsilon_i, \quad \text{for all } i = 1, \dots, m.$$
(15)

Furthermore, assuming conditional mean independence [Eq. (20)] implies uncorrelated measurement errors, i.e.,

$$Cov(\varepsilon_i, \varepsilon_j) = 0, \quad i \neq j, \quad \text{for all } i, j = 1, \dots, m.$$
 (16)

Typical models of CTT consist of combining one of the equivalence assumptions with the conditional mean assumption, and in some models we additionally assume equal error variances [see Eq. (21)]. For example, the most restrictive model, the model of parallel variables, consists of Equations (17), (20), and (21). The model of essentially τ -equivalent variables consists of Equations (18) and (20), and the least restrictive model, the model of *congeneric variables*, is defined by Equations (19) and (20). Note that, in empirical applications, all these assumptions can be wrong. However, in principle, and in contrast to the properties listed in Table 2, they are also empirically testable, because they imply a certain structure of the expectations, variances and covariances of the observables Y_i in the total population, but also in and

(9)

(10)

 Table 3.
 Assumptions defining various CTT models

Equivalence assumptions

For $\tau := \tau_1$ and $\lambda_{i0}, \lambda_{i1} \in \mathbb{R}$:

 $\tau_i = \tau$, for all i = 1, ..., m, (τ -equivalence) (17)

 $\tau_i = \lambda_{i0} + \tau$, for all i = 1, ..., m, (essential τ -equivalence) (18)

 $\tau_i = \lambda_{i0} + \lambda_{i1} \cdot \tau, \quad \text{for all } i = 1, \dots, m, \quad (\tau\text{-congenericity}). \tag{19}$

Conditional mean independence

 $E(Y_i | U, Y_1, \dots, Y_{i-1}, Y_{i+1}, \dots, Y_m) = E(Y_i | U).$ ⁽²⁰⁾

Equal variances of measurement error variables

$$Var(\varepsilon_i) = Var(\varepsilon_i).$$
 (21)

Note. These equations are assumed to hold for all i, j = 1, ..., m. A model of CTT consists of one of the equivalence assumptions and conditional mean independence. Additionally assuming equal error variances is optional.



Figure 1. True score and error variables for three manifest variables Y_1 , Y_2 , and Y_3 .

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Figure 2. The model of congeneric variables for three manifest variables *Y*₁, *Y*₂, and *Y*₃.

between subpopulations.¹ For more details see ? (?) or ? (?). All these implications can easily be tested in structural equation modeling.

2.6 Limitations of CTT

As shown above, the CTT concept of a trait (the latent variable τ) has a mathematical structure that allows to derive its properties (see, e.g., Table 2) and that guides, but also limits its interpretation. As mentioned before, in all models of CTT, the common latent variable τ is a (deterministic) function of each of the true-score variables. Therefore, it is also a function of the person variable U, and this means that each of its values characterize a person.

The Person-in-a-Situation

The construction of the latent variable τ in classical test theory as outlined above has been questioned arguing that a person can never be assessed in a situational vacuum (cf. Steyer, Ferring, & Schmitt, 1992, p. 96). In other words, at the time of measurement, we do not condition on a person *u*, but on a "person-in-a-situation" (Anastasi, 1983).

This argument challenges the substantive interpretation of the person variable U and the interpretation of the latent variable τ even if we consider only one single occasion of measurement. If the argument is correct, then a value u of U in CTT is not a person, but a *person-in-the-situation* in which the measurement is made, i.e., $u = (u_0, s)$ and $U = (U_0, S)$, where U_0 represents the person variable and S the situation variable pertaining to the occasion of measurement considered. Hence, instead of defining τ_i by Equation (5), we should rather define

$$\tau_i := E(Y_i | U_0, S), \tag{22}$$

where *S* represents the situation variable with values $s_1, s_2, ...,$ each representing a situation in which a person could be, at the occasion of measurement considered. According to this definition, τ_i is the conditional expectation of the observable Y_i given the person variable U_0 and the situation variable *S*. Hence, τ_i is a random variable with values $E(Y_i | U_0 = u_0, S = s)$, the conditional expectation given person u_0 and situation *s*.

¹If we use the term 'total population' we refer to the random experiment described in section 2.1, in which we sample a person from Ω_U . In contrast, the term 'subpopulation' refers the same random experiment except for sampling from a subset of Ω_U , such as the subset of male persons in Ω_U or the subset of female persons in Ω_U .

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Consequently, the latent variable τ in the model of τ -congeneric variables, which is an (arbitrarily fixed deterministic) positive linear function of each τ_i , does not represent a property of a person, i.e., a trait, instead it represents a property of a person-in-a-situation, i.e., a state that is affected by the person and by the situation in which the measurement is made, as well as by the interaction between person and situation.

The methodological implication is that the latent variable τ in a CTT model consists of a trait component and a situational/interactional component. Furthermore, in a cross-sectional design, it is neither possible to disentangle τ itself nor its variance into these two components. In other words, in a cross-sectional design there is no way to determine that part of *Var*(τ) which is due to *situational and/or interactional fluctuations* and that part which is due to the *trait* to be measured.²

The Person at Time t

Anastasi's critique of the CTT concept of a trait that motivated the early versions of LST theory (see, e.g., Steyer et al., 1992) has to be complemented if several occasions of measurement are considered. If we measure a person at two occasions of measurement, then we measure a person u_0 in situation s_1 at occasion 1. However, what about occasion 2? Do we actually measure the same person u_0 in situation s_2 at time 2? What about the *experiences* of the person in between time 1 and 2? Is it not true that the complete past of the process considered in a longitudinal study makes up the *person at time t*? This past with respect to time 2 does not only include the experiences in between times 1 and 2 but also the values of the manifest variables assessed at time 1 and the situations that realized at time 1. If I agree to an item saying that I am depressed at time 1 then this can have an effect on my depression at time 2. Similarly, the situation that I did not pass an important examination at or briefly before time 1 may determine my depression also at time 2. In other words, which person I am at time 2 depends on the complete past with all experiences I made, all situations that realized at previous times, and all manifest variables scores assessed in the past. Hence, we should not only distinguish between variable states and invariant traits. Instead traits should be considered changeable and even the concept of a person should be dynamic reflecting both invariance and change of the person considered. Simply speaking, we should distinguish between John Smith at time 1 and the "same" John Smith at time 2. In a sense, John is the same and yet a different person at time 2, and this should be reflected in a theory of states and traits.

3 Construction of Latent Variables in LST Theory

3.1 The Kind of Random Experiments and Their Mathematical Representation

In latent state-trait (LST) theory, we consider the following kind of empirical phenomenon:

(a) A person u_0 is sampled from a set Ω_{U_0} of persons.

²Similarly, in CCT we cannot disentangle measurement error and true score variance with a single measurement *Y*. This is why we need several measures Y_i in order to identify the variances of τ and ε_i , so that we know which part of $Var(Y_i) = Var(\tau) + Var(\varepsilon_i)$ is due to τ and ε_i , respectively.

- (b) The person u_0 will make experiences e_1 before assessing the manifest variables at time 1.
- (c) The person u_0 will be in a situation s_1 at the time of assessing the manifest variables at time 1.
- (d) Behavior o_1 is observed.
- (e) Items (b) to (d) are repeated for times *t* = 2,..., *n* and for these times of measurement, *t* replaces the index 1 above.

The observation o_t is an element of the set Ω_{O_t} of possible observations at time t. This set is a set product $\Omega_{O_t} = \Omega_{O_{1t}} \times \ldots \times \Omega_{O_{mt}}$ of m sets of possible observations at time t. For example, if we consider just two real-valued observations — such as the score of the person on two depression scales, then $\Omega_{O_t} = \Omega_{O_{1t}} \times \Omega_{O_{2t}} = \mathbb{R} \times \mathbb{R}$, and the observation o_t is a pair of two real numbers, i.e., $o_t = (a, b)$, where $a, b \in \mathbb{R}$.

The kind of empirical phenomenon described by (a) to (e) is a *random experiment* with the set Ω of possible outcomes specified in Equation (23) of Table 4. If, for simplicity, we just consider *n* = 2 times of assessment, then the elements of Ω are tuples such as

$$\omega = (u_0, e_1, s_1, o_1, e_2, s_2, o_2).$$

We choose an appropriate set of events, a σ -*algebra* \mathcal{A} of subsets of Ω . Furthermore, we assume that there is a (usually unknown) *probability measure* P on \mathcal{A} . The probability space (Ω , \mathcal{A} , P) is the mathematical representation — and describes the mathematical structure — of the random experiment considered.

3.2 The Primitives of LST Theory

The Person at Time t and the Person Variables U_t

Consider the projection mapping $U_1: \Omega \to \Omega_{U_0} \times \Omega_{E_1}$ [see Eq. (24)], which maps each possible outcome $\omega \in \Omega$ into the set $\Omega_{U_0} \times \Omega_{E_1}$. That is, if the random experiment is actually conducted and the person u_0 is sampled, making the experiences e_1 before time 1 of assessment, then a value of U_1 is $u_1 = U_1(\omega) = (u_0, e_1)$, and this pair (u_0, e_1) represents the *person at time* 1. It differs from u_0 , the person at the time it is sampled, by the experiences e_1 made between sampling and assessment at t = 1. The actual experiences made could have been different, and this is why e_1 is just one element of a set Ω_{E_1} of possible experiences than can be made before this time point.

Similarly, for t = 2, consider the projection mapping U_2 [see Eq. (25)], the *person variable at time* 2, and S_2 [see Eq. (26)], the *situation variables at time* 2. The person variable U_2 is again a projection, mapping each possible outcome $\omega \in \Omega$ onto the set $\Omega_{U_0} \times \Omega_{E_1} \times \Omega_{S_1} \times \Omega_{O_1} \times \Omega_{E_2}$. If the random experiment is actually conducted and the person u_0 is sampled making the experiences e_1 , being in situation s_1 at time 1, yielding observation o_1 , and making the experiences e_2 before assessment at time 2, then a value $U_2(\omega)$ of the person variable U_2 is $u_2 = (u_0, e_1, s_1, o_1, e_2)$, and this quintuple represents the person at time 2. Comparing $u_1 = (u_0, e_1)$ to $u_2 = (u_0, e_1, s_1, o_1, e_2)$ shows how it is possible to refer to same person (u_0, e_1) and still have a dynamic concept of a person. The person at time 2 (u_2) shares (u_0, e_1) with u_1 but differs from u_1 by (s_1, o_1, e_2) , i.e., it differs from u_1 the situation s_1 that actually realizes at time 1, the observation o_1 made time 1 and the experiences e_2 made between time 1 and time 2 of assessment. Note that (Ω, \mathcal{A}, P) is assumed to be — and it always can be — constructed such that the person variables U_t are (nonnumerical) *random variables*.

Table 4.	Basic concepts of LST theory

The set of possible outcomes of the random experiment

$$\Omega = \Omega_{U_0} \times \Omega_{E_1} \times \Omega_{S_1} \times \Omega_{O_1} \times \ldots \times \Omega_{E_t} \times \Omega_{S_t} \times \Omega_{O_t} \times \ldots \times \Omega_{E_n} \times \Omega_{S_n} \times \Omega_{O_n}$$
(23)

Primitives of LST theory

Person variable at time $t = 1$	$U_1: \Omega \to \Omega_{U_0} \times \Omega_{E_1}$	(24)
Person variable at times $t > 1$	$U_t \colon \Omega \to \Omega_{U_0} \times \Omega_{E_1} \times \Omega_{S_1} \times \Omega_{O_1} \times \ldots \times \Omega_{E_t}$	(25)
Situation variable at time t	$S_t: \Omega \to \Omega_{S_t}$	(26)
Manifest random variables	$Y_{it}: \Omega \to \mathbb{R}$	(27)
(U_t, S_t) -Conditional expectations	$E(Y_{it} U_t, S_t) \colon \Omega \to \mathbb{R}$	(28)
U_t -Conditional expectations	$E(Y_{it} U_t) \colon \Omega \to \mathbb{R}$	(29)

Theoretical variables of LST theory

Latent state variables	$\tau_{it} := E(Y_{it} U_t, S_t)$	(30)
Measurement error variables	$\varepsilon_{it} := Y_{it} - \tau_{it}$	(31)
Latent trait variables	$\xi_{it} := E(Y_{it} U_t)$	(32)
Latent state residuals	$\zeta_{it} := \tau_{it} - \xi_{it}$	(33)

Important coefficients of LST theory

Reliability	$Rel(Y_{it}) := Var(\tau_{it}) / Var(Y_{it})$	(34)
Consistency	$Con(Y_{it}) := Var(\xi_{it})/Var(Y_{it})$	(35)

Occasion specificity
$$Spe(Y_{it}) := Var(\zeta_{it})/Var(Y_{it})$$
 (36)

Note that the definition of the person variables U_t described above has been changed in the new version of LST theory presented in this paper, and this has implications, in particular for the definition of the latent trait variables (see Table 4). Now the concept of a person variable U_t allows for the persons to *change* between different times of measurement. In contrast, although persons could be in different situations at different occasions of measurement, in previous versions of the theory we only considered a static person variable U_t .

The Situations at Time t and the Situation Variables S_t

As mentioned before, whenever a person is assessed (measured, tested, observed, rated), he or she is a specific situation. There is no situational vacuum. By situation we mean everything that might be relevant for the result of the assessment. This includes psychological situations (such as being in a specific mood state), physical situations (such as being exhausted), social situations (such as being in a group or being alone), biological situations (such as being hungry or full), etc., and the combination of all such situations. Whatever the situation is, we can consider it to be an element of set of situations Ω_{S_t} , one of which realizes at the time *t*. Therefore, we consider the projection mappings S_t that map the possible outcome $\omega \in \Omega$ of the



Figure 3. Latent states, latent traits, measurement errors, and latent state-trait residuals.

random experiment onto the set Ω_{S_t} . Such a *situation variable* S_t is again a (nonnumerical) random variable, just like the person variables U_t discussed above.

The Manifest Variables Y_{it} and Their Conditional Expectations

The (manifest) random variables Y_{it} have two indices referring to measurement i at time t. According to the specific scoring rule for measurement i (at time t), each of these manifest variables Y_{it} maps each possible outcome $\omega \in \Omega$ into the set \mathbb{R} of real numbers. Assuming that the Y_{it} are nonnegative or have finite expectations, we can consider their conditional expectations, which again are random variables on (Ω, \mathcal{A}, P) .

Using the person variables U_t and the situation variables S_t , we consider the (U_t, S_t) -conditional expectations of Y_{it} with values

$$E(Y_{it} | U_t, S_t)(\omega) = E(Y_{it} | U_t = u_t, S_t = s_t), \quad \text{if } (U_t, S_t)(\omega) = (u_t, s_t). \tag{37}$$

Hence, the values of such a (U_t, S_t) -conditional expectation $E(Y_{it} | U_t, S_t)$ are the $(U_t = u_t, S_t = s_t)$ -conditional expectations $E(Y_{it} | U_t = u_t, S_t = s_t)$. Similarly, we also consider the U_t -conditional expectations of Y_{it} with values

$$E(Y_{it}|U_t)(\omega) = E(Y_{it}|U_t = u_t), \quad \text{if } U_t(\omega) = u_t. \tag{38}$$

This shows that the values of a U_t -conditional expectation $E(Y_{it}|U_t)$ are the $(U_t=u_t)$ conditional expectations $E(Y_{it}|U_t=u_t)$. The conditional expectations specified in Equations (37) and (38) will be used in the next section to define the basic concepts of LST theory.

The probability space (Ω, \mathcal{A}, P) and the random variables U_t , S_t , Y_{it} , $E(Y_{it} | U_t, S_t)$, and $E(Y_{it} | U_t)$, i = 1, ..., m, t = 1, ..., n, are the primitives used to define the basic concepts of LST theory.

3.3 Basic Concepts of LST Theory

In the next part of Table 4 we define the *latent state variables* τ_{it} , the *measurement error variables* ε_{it} , the *latent trait variables* ξ_{it} , and the *latent state residual* of ζ_{it} [see Eqs. (30) to (33)]. These are the four basic concepts of LST theory. Note that

$$\zeta_{it} = \tau_{it} - \xi_{it} = \tau_{it} - E(Y_{it} | U_t) = \tau_{it} - E(\tau_{it} | U_t),$$
(39)

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because

$$E(\tau_{it}|U_t) = E[E(Y_{it}|U_t, S_t)|U_t] = E(Y_{it}|U_t) = \xi_{it}.$$
(40)

Equation (39) shows that ζ_{it} is a *residual* of τ_{it} with respect to the regressor U_t . (See chapter 10 of Steyer, Nagel, Partchev, & Mayer, in press, for the properties of residuals with respect to a conditional expectation.)

LST Coefficients

The last part of Table 4 displays the most important coefficients of LST theory. The reliability coefficient $Rel(Y_{it})$ quantifies how well, on average, the manifest variable Y_{it} estimates the latent state variable τ_{it} . Similarly, the consistency coefficient $Con(Y_{it})$ quantifies how well, on average, the manifest variable Y_{it} estimates the latent trait variable ξ_{it} . In contrast, the occasion specificity coefficient $Spe(Y_{it})$ quantifies which proportion of the variance of the manifest variable Y_{it} is determined by the latent state residual ζ_{it} that represents situation effects and the effects of the interaction between person and situation on Y_{it} .

Implications of the Definitions

The fact that the variables ζ_{it} are residuals with respect to a conditional expectation implies some of their properties displayed in Table 5. To emphasize, these and the other properties displayed in this table are solely implied by the definitions of the variables involved. All these properties are *always true* provided that the expectations $E(Y_{it})$ and the variances $Var(Y_{it})$, i = 1, ..., m, t = 1, ..., n, are finite. They can be derived from the definitions of the four LST-theoretical variables displayed in Table 4.

In Figure 1 some of these properties are represented in a path diagram. The most important ones are:

- (a) Each manifest variable Y_{it} has its own latent trait variable ξ_{it} , its own latent state variable τ_{it} , its own measurement error variable ε_{it} , and its own latent state residual ζ_{it} .
- (b) Latent trait variable variables may correlate with each other.
- (c) Latent state variable variables may correlate with each other and with the latent trait variables.
- (d) Measurement error variables may correlate with each other.

Furthermore, if $s \le t$, then

- (e) Measurement error variables ε_{it} on one side and latent state residuals ζ_{js} on the other side are uncorrelated [see Table 5, Eq. (49)].
- (f) Measurement error variables ε_{it} on one side and latent state variables τ_{js} on the other side are uncorrelated [see Table 5, Eq. (50)].
- (g) Measurement error variables ε_{it} on one side and latent trait variables ξ_{js} on the other side are uncorrelated [see Table 5, Eq. (51)].
- (h) Latent state-residuals on one side and latent trait variables on the other side are uncorrelated [see Table 5, Eq. (52)].

Finally,

Table 5. Some properties of the basic concepts of LST theory

Decomposition of variables

$$\tau_{it} = \xi_{it} + \zeta_{it} \tag{41}$$

$$Y_{it} = \tau_{it} + \varepsilon_{it} \tag{42}$$

$$=\xi_{it}+\zeta_{it}+\varepsilon_{it} \tag{43}$$

Decomposition of variances

1

$$Var(\tau_{it}) = Var(\xi_{it}) + Var(\zeta_{it})$$
(44)

$$Var(Y_{it}) = Var(\tau_{it}) + Var(\varepsilon_{it})$$
 (45)

$$= Var(\xi_{it}) + Var(\zeta_{it}) + Var(\varepsilon_{it})$$
(46)

Other properties

$E(\varepsilon_{it}) = 0$		(47)
$E(\zeta_{it}) = 0$		(48)
$Cov(\varepsilon_{it},\zeta_{js}) = 0,$	$s \le t$	(49)
$Cov(\varepsilon_{it},\tau_{js}) = 0,$	$s \le t$	(50)
$Cov(\varepsilon_{it},\xi_{js}) = 0,$	$s \le t$	(51)
$Cov(\zeta_{it},\xi_{js}) = 0,$	$s \le t$	(52)
$Cov(\zeta_{it},\zeta_{js}) = 0,$	$s \neq t$	(53)
$E(\varepsilon_{it} U_s, S_t) = 0,$	$s \le t$	(54)
$E(\varepsilon_{it} U_s) = 0,$	$s \le t$	(55)
$E(\zeta_{it} U_s) = 0,$	$s \le t$	(56)
$\mathcal{E}(\varepsilon_{it} \xi_s, \tau_{is}) = 0,$	$s \le t$	(57)
$E(\zeta_{it} \xi_s)=0,$	$s \le t$	(58)
$Rel(Y_{it}) = Con$	$n(Y_{it}) + Spe(Y_{it}).$	(59)

Note. All properties in this table hold for all $i, j \in I := \{1, ..., m\}$, $s, t \in T := \{1, ..., n\}$, provided that the expectations $E(Y_{it})$ and the variances $Var(Y_{it})$ are finite. All these properties (and many others) follow from the definition of latent trait, latent state, measurement error variables, and latent state residuals.

(i) Latent state-residuals pertaining to different time points are uncorrelated [see Table 5, Eq. (53)].

These propositions about the covariances follow from the definitions of the latent state and latent trait variables [see Table 4, Eqs. (30) to (33)] using the fact that residuals are uncorrelated with functions of their regressors (for more details, see Steyer et al., in press, ch. 10).

To emphasize, this equation and all properties displayed in Table 5 cannot be wrong in any empirical application. These properties are always true, in the same sense that it is always true that a bachelor is unmarried. (Of course, you may find that a purported bachelor is married. However, then we can conclude that he actually is not a bachelor. We can not conclude that a bachelor can be married.) In contrast, the assumptions defining models of LST theory, which will be introduced in the following section, *can* be wrong in empirical applications.

3.4 Models of LST Theory

Models of LST theory are defined by assumptions about the LST-theoretical concepts. Different assumptions define different models, only some of which will be discussed in this paper in more detail.

Multistate Models

For the latent state variables we define, for all t = 1, ..., n,

$$\tau_t := \tau_{1t} \tag{60}$$

and make one of the *equivalence assumptions* displayed in Equations (61) to (63) (see Table 6). Each of these assumptions is a different way of saying that within each time point *t*, the manifest variables Y_{it} measure the same latent state variable, denoted τ_t . Therefore, if one of these three assumptions holds, then τ_t is also called a *common latent state variable*.

Of course, defining $\tau_t := \tau_{1t}$ in the model of essentially τ_t -equivalent variables is arbitrary, because $\tau_t^* := \alpha + \tau_{1t}$, for any real number α would do as well. Hence, the common latent state variable τ_t is uniquely defined only up to translations, i.e., if τ_t and τ_t^* are two versions of the common latent state variable, then there is a real number α such that $\tau_t = \tau_t^* + \alpha$.

Similarly, in the model of τ_t -congeneric variables, the common latent state variable τ_t could also be defined by $\tau_t := \alpha + \beta \cdot \tau_{1t}$, for any pair of real numbers α and β . Nevertheless, in this model, the common latent state variable τ_t is uniquely defined up to linear transformations.

Next, we add the assumption of *conditional mean independence*: the expectations of the manifest variables Y_{it} only dependent on the person variable U_t and the situation variable S_t , but not additionally on person variables or situation variables pertaining to other time points nor on other manifest variables Y_{is} , $(js) \neq (it)$.

Finally, if *scale invariance over time* is desired for the model of essentially τ_t -equivalent variables, we may additionally assume

$$\lambda_{it0} = \lambda_{is0} \tag{65}$$

for essential τ_t -equivalence, and

$$\lambda_{it0} = \lambda_{is0} \quad \text{and} \quad \lambda_{it1} = \lambda_{is1}$$
 (66)

Table 6. Assumptions defining various kinds of multistate models

Equivalence assumptions

For $\tau_t := \tau_{1t}$ and $\lambda_{it0}, \lambda_{it1} \in \mathbb{R}$:

$\tau_{it} = \tau_t$		$(\tau_t$ -equivalence)	(61)
$\tau_{it} = \lambda_{it0} + \tau_t$, when	$e \lambda_{1t0} = 0 \qquad (essen$	tial τ_t -equivalence)	(62)
$\tau_{it} = \lambda_{it0} + \lambda_{it1} \cdot \tau_t,$	where $\lambda_{1t0} = 0$ and $\lambda_{1t1} = 1$,	$(\tau_t$ -congenericity)	(63)

Conditional mean independence

$$E[Y_{it} | U_1, \dots, U_n, S_1, \dots, S_n, (Y_{js}, (j, s) \in I \times T \setminus \{(i, t)\})] = E(Y_{it} | U_t, S_t).$$
(64)

Note. These equations are assumed to hold for all $i, j \in I := \{1, ..., m\}$, $s, t \in T := \{1, ..., n\}$. A multistate model consists of one of the equivalence assumptions and conditional mean independence.

for τ_t -congenericity), all equations for all i = 1, ..., m, and s, t = 1, ..., n.

The upper path diagram of Figure 4 shows such a multistate model with τ_t congenericity. In the other parts of this figure we already utilize some of the implications of the assumptions of a multistate model, which are discussed in the following
section.

Implications

Each of the assumptions (61) to (63) in Table 6 has a number of implications. Some of these implications concern the latent trait variables ξ_{it} within each of the time points *t*. For all multistate models we defined $\tau_t := \tau_{1t}$ (see Table 6). This implies, for all t = 1, ..., n,

$$\xi_{1t} = E(Y_{1t} | U_t) \qquad [\text{see Eq. (32)}] \\ = E(\tau_{1t} | U_t) \qquad [\text{see Eq. (40)}] \\ = E(\tau_t | U_t) \qquad [\tau_t := \tau_{1t}].$$
(67)

Now, for all t = 1, ..., n, we also define

$$\xi_t := \xi_{1t},\tag{68}$$

the *common latent trait variable* within time point *t*.

Assuming τ_t -equivalence within each time point [see Eq. (61) in Table 6] implies, for all i = 1, ..., m and t = 1, ..., n,

$$\xi_{it} = E(Y_{it} | U_t) = E(\tau_{it} | U_t)$$
 [see Eq. (40)]
= $E(\tau_t | U_t)$ [see Eq. (61)] (69)
= ξ_t [see Eqs. (67), (68)].

Hence, assuming τ_t -equivalence implies ξ_t -equivalence within each time point [see Eq. (71) in Table 7].



Figure 4. Three equivalent path diagrams of a multistate model. In such a model the variances $Var(\xi_t)$ and $Var(\zeta_t)$, t = 1, 2, are not identified.

Similarly, assuming essential τ_t -equivalence [Eq. (62) in Table 6] implies, for all i = 1, ..., m and t = 1, ..., n,

$$\xi_{it} = E(Y_{it} | U_t) = E(\tau_{it} | U_t)$$
 [see Eq. (40)]
= $E(\lambda_{it0} + \tau_t | U_t)$ [see Eq. (62)]
= $\lambda_{it0} + E(\tau_t | U_t)$
= $\lambda_{it0} + \xi_t$ [see Eqs. (67), (68)]. (70)

Hence, within each time *t*, the Y_{it} -specific latent traits ξ_{it} differ only with respect to the additive constant λ_{it0} . In other words, assuming essential τ_t -equivalence within each time point [see Eq. (62) in Table 6] implies *essential* ξ_t -*equivalence* within each time point [see Eq. (72) in Table 7]. In this sense, the latent trait variable ξ_t is common to all variables Y_{it} within time point *t*.

Analogously, it is easily seen: Assuming τ_t -congenericity within time points [see Eq. (63)] implies ξ_t -congenericity within time points [see Equation (73) in Table 7]. The lower two path diagrams in Figure 4 show the congeneric version of a multistate model, and these path diagrams include the common latent trait variables ξ_t . Note, however, that in such a model the variances $Var(\xi_t)$ and $Var(\zeta_t)$, t = 1, 2, are not identified.

Other implications of the multistate model are related to the *latent state residuals*. If we assume τ_t -equivalence, then Equation (61) implies

$$\zeta_{it} = \tau_{it} - \xi_{it} = \tau_t - \xi_t.$$
(85)

Similarly, Equation (62) implies

$$\zeta_{it} = \tau_{it} - \xi_{it} = \lambda_{it0} + \tau_t - (\lambda_{it0} + \xi_t) = \tau_t - \xi_t.$$
(86)

Table 7. Implications of multistate models

Implications of the equivalence assumptions

(61)	\Rightarrow	$\xi_{it} = \xi_t$	$(\xi_t$ -equivalence)	(71)
(62)	⇒	$\xi_{it} = \lambda_{it0} + \xi_t$	(essential ξ_t -equivalence)	(72)
(63)	⇒	$\xi_{it} = \lambda_{it0} + \lambda_{it1} \cdot \xi_i$	t (ξ_t -congenericity)	(73)
$[(61) \lor (62) \lor (63)]$	⇒	$\tau_t = \xi_t + \zeta_t$		(74)
		$Var(\tau_t) = Var(\xi_t) +$	$Var(\zeta_t)$	(75)
$(61) \lor (62)$	⇒	$\zeta_{it} = \zeta_t$	$(\zeta_t$ -equivalence)	(76)
		$Var(Y_{it}) = Var(\tau_t) +$	$Var(\varepsilon_{it})$	(77)
(63)	⇒	$\zeta_{it} = \lambda_{it1} \cdot \zeta_t$	$(\zeta_t$ -congenericity)	(78)
		$Var(Y_{it}) = \lambda_{it1}^2 \cdot Var$	$(\tau_t) + Var(\varepsilon_{it}).$	(79)

Implications of conditional mean independence on covariances

(64)
$$\Rightarrow Cov(\varepsilon_{it}, \varepsilon_{js}) = 0, \quad (i, t) \neq (j, s)$$
 (80)

$$[(61) \lor (62) \lor (63)] \land (64) \implies Cov(\zeta_t, \xi_s) = 0$$
(81)

~ ()

$$Cov(\varepsilon_{it}, \tau_s) = 0 \tag{82}$$

$$Cov(\varepsilon_{it},\zeta_s) = 0$$

$$Cov(\varepsilon_{it},\zeta_s) = 0.$$
(84)

$$ov(\varepsilon_{it},\zeta_s) = 0. \tag{84}$$

Note. We define $\tau_t := \tau_{1t}$, $\xi_t := E(\tau_t | U_t)$, and $\zeta_t := \tau_t - \xi_t$. All equations hold for all i, j = 1, ..., m and s, t = 1, ..., n.

3 CONSTRUCTION OF LATENT VARIABLES IN LST THEORY

Finally, assuming τ_t -congenericity [see Eq. (63)] implies

$$\zeta_{it} = \tau_{it} - \xi_{it} = \lambda_{it0} + \lambda_{it1} \tau_t - (\lambda_{it0} + \lambda_{it1} \xi_t) = \lambda_{it1} \cdot (\tau_t - \xi_t).$$
(87)

This means that, in each of these multistate models, we can define

$$\zeta_t := \tau_t - \xi_t, \tag{88}$$

which yields Equation (78) (see Table 7).

The path diagram in the lower left-hand side of Figure 4 visualizes these implications of the multistate model of τ_t -congeneric variables for the manifest variables Y_{it} , namely,

$$Y_{it} = \lambda_{it0} + \lambda_{it1} \cdot \tau_t + \varepsilon_{it} \tag{89}$$

and

$$\tau_t = \xi_t + \zeta_t. \tag{90}$$

Hence, in the multistate model, each latent state variable τ_t has its own latent trait variable ξ_t and its own latent state residual ζ_t . Although each of the three equivalence assumptions implies $Var(\tau_t) = Var(\xi_t) + Var(\zeta_t)$, the variances $Var(\xi_t)$ and $Var(\zeta_t)$ cannot be separated from their sum, $Var(\tau_t)$, unless additional assumptions are introduced, i.e., the variances $Var(\xi_t)$ and $Var(\zeta_t)$ are not identified.

The path diagram in the lower right-hand side of Figure 4 is an alternative visualization of the implications of the multistate model of τ_t -congeneric variables. This path diagram results from substituting Equation (90) into Equation (89), i.e.,

$$Y_{it} = \lambda_{it0} + \lambda_{it1} \cdot \tau_t + \varepsilon_{it}$$

= $\lambda_{it0} + \lambda_{it1} \cdot (\xi_t + \zeta_t) + \varepsilon_{it}$
= $\lambda_{it0} + \lambda_{it1} \cdot \xi_t + \lambda_{it1} \cdot \zeta_t + \varepsilon_{it}.$ (91)

For simplicity, the intercepts λ_{it0} are not represented in the diagram.

3.5 Multistate-Singletrait Models

The first class of models that allows for the identification of the variances and covariances of the latent state variables, the latent trait variables, and the latent state residuals is the multistate-singletrait model. This model is defined by adding one of the three assumptions presented in Table 8 to the assumptions defining a multistate model. With θ -equivalence [see Table 6, Eq. (92)] we assume that there is no trait change at all over all *n* occasions of measurement. The latent trait variable θ is identical to the time-specific latent trait variables ξ_t , which themselves are identical to the Y_{it} -specific latent trait variables ξ_{it} .

In contrast, with *essential* θ -*equivalence* [see Table 6, Eq. (93)] trait change is possible, but we assume that the amount of change from one time point to another one is the same for each and every person in the population, i.e., for all persons that might be sampled in the random experiment considered. In other words, trait change can perfectly be described by a translation, i.e., by adding a constant that is identical for all persons in the population.

Finally, with θ -congenericity [see Table 6, Eq. (94)] we also allow for trait change, but it is assumed that this change can perfectly be described by a linear function

Table 8. Additional assumptions and implications of multistate-singletrait models

Additional assumptions of the multistate-singletrait model

For $\theta := \xi_1$ and $\lambda_{t0}, \lambda_{t1} \in \mathbb{R}$:

$\xi_t = \theta,$	$(\theta$ -equivalence)	(92)
$\xi_t = \lambda_{t0} + \theta$	(essential θ-equivalence)	(93)
$\xi_t = \lambda_{t0} + \lambda_{t1} \cdot \theta$	$(\theta$ -congenericity).	(94)

Additional implications on variables and their variances

$[(61) \lor (62) \lor (63)] \land (92) \ \Rightarrow$	$\tau_t = \theta + \zeta_t$	(95)
	$Var(\tau_t) = Var(\theta) + Var(\zeta_t)$	(96)

$$[(61) \lor (62) \lor (63)] \land (93) \Rightarrow \tau_t = \lambda_{t0} + \theta + \zeta_t \tag{97}$$

$$Var(\tau_t) = Var(\theta) + Var(\zeta_t)$$
(98)

$$[(61) \lor (62) \lor (63)] \land (94) \Rightarrow \tau_t = \lambda_{t0} + \lambda_{t1} \cdot \theta + \zeta_t$$
(99)

$$Var(\tau_t) = \lambda_{t1}^2 \cdot Var(\theta) + Var(\zeta_t)$$
(100)

Additional implications on covariances

$$Cov(\varepsilon_{it},\theta) = 0 \tag{101}$$

$$Cov\left(\zeta_t,\theta\right) = 0. \tag{102}$$

Note. One of the three equivalence assumptions [Eqs. (92) to (94)] is made additional to the assumptions defining a multistate model (see Table 6). The variables ξ_t are defined by Equation (68). All equations are assumed to hold for all i = 1, ..., m and t = 1, ..., n.

with the same two coefficients (intercept and slope) for all persons, implying that the correlation between the latent trait variables ξ_s and ξ_t is 1, for all s, t = 1, ..., n.

The most important *additional implications of the multistate-singletrait model* are also presented in Table 8. Note that these implications are *additional* to the implications already following from the multistate model (see Table 7). These additional implications concern the *common latent state variables* τ_t and their variances [see Eqs. (96), (98), (100)], and the covariances of the *common latent trait variable* θ with measurement errors and with *common latent state residuals* [see Eqs. (101), (102)].

To emphasize, all implications of the definitions of latent state and trait variables and of the multistate models displayed in Tables 7 and 5 are still true in the corresponding multistate-singletrait model. For example, if we assume τ_t -congenericity [Eq. (63)] and θ -congenericity [Eq. (94)], then the equations

$$Y_{it} = \tau_{it} + \varepsilon_{it}$$

$$= (\lambda_{it0} + \lambda_{it1} \cdot \tau_t) + \varepsilon_{it}$$

$$= \lambda_{it0} + \lambda_{it1} \cdot (\lambda_{t0} + \lambda_{t1} \cdot \theta + \zeta_t) + \varepsilon_{it}$$

$$= (\lambda_{it0} + \lambda_{it1}\lambda_{t0} + \lambda_{it1}\lambda_{t1} \cdot \theta) + \lambda_{it1} \cdot \zeta_t + \varepsilon_{it}$$

$$= \xi_{it} + \zeta_{it} + \varepsilon_{it}$$
[(43)]



Figure 5. A congeneric multistate-singletrait model for two observations at each of two occasions of measurement

hold for the manifest variables Y_{it} . Correspondingly,

$$Var(Y_{it}) = Var(\tau_{it}) + Var(\varepsilon_{it})$$

$$= \lambda_{it1}^{2} Var(\tau_{t}) + Var(\varepsilon_{it})$$

$$= \lambda_{it1}^{2} \lambda_{t1}^{2} Var(\theta) + \lambda_{it1}^{2} Var(\zeta_{t}) + Var(\varepsilon_{it})$$

$$= Var(\xi_{it}) + Var(\zeta_{it}) + Var(\varepsilon_{it})$$

$$[(104)]$$

$$= Var(\xi_{it}) + Var(\zeta_{it}) + Var(\varepsilon_{it})$$

$$[(46)]$$

hold for the variances of the Y_{it} .

To summarize: Two of the multistate-singletrait models allow for trait change. However, under essential τ_t -equivalence (the first of these two), trait change is restricted to changes that can be described by adding the same constant to the trait scores of all persons. Trait change under τ_t -congenericity allows for changes that can be described by adding a constant and multiplying by another constant, where again the two constants are identical for all persons.

3.6 Multistate-Multitrait Models

Multistate-multitrait models allow for more complex kinds of trait change. Table 9 displays the assumptions that are made additionally to the assumptions defining a multistate model (see Table 6). For simplicity, these additional assumptions are presented for the case of two traits. The first of these two traits, θ_1 , is measured at n_1 time points, the second one, θ_2 , at n_2 time points. Figure 6 shows the path diagram of the congeneric version of the model for $n_1 = n_2 = 2$ time points.

Note that the latent trait variable θ_1 is defined by $\theta_1 := \xi_1$ (see Table 9). This means that, in Figure 6, there are no residuals for xi_1 and xi_2 . Remember, ξ_1 has been defined by $\xi_1 := E(\tau_t | U_t)$ [see Eq. (68)]. Tracing back all these definitions yields

$$\theta_1 = \xi_1 = \xi_{11}. \tag{113}$$

Hence, in the model represented in Figure 6, the latent trait variable θ_1 is defined such that it is identical to the Y_{11} -specific latent trait variable ξ_{11} . Correspondingly, in this model,

$$\theta_2 = \xi_3 = \xi_{13}. \tag{114}$$

In contrast to the multistate-singletrait model, the multistate-multitrait model allows for *any kind of trait change*. Usually, the two latent trait variables θ_1 and θ_2

 Table 9.
 Additional assumptions and implications of multistate-doubletrait-models

For $\theta_1 := \xi_1$, $\theta_2 := \xi_{n_1+1}$, and λ_{t0} , $\lambda_{t1} \in \mathbb{R}$:

$\xi_t = \theta_1$, for $t = 2, \dots, n_1$	(θ_1 -equivalence)	(105)
$\xi_t = \theta_2$, for $t = n_1 + 2, \dots, n_2$	$(\theta_2$ -equivalence)	(106)
$\begin{aligned} \xi_t &= \lambda_{t0} + \theta_1, \text{for } t = 2, \dots, n_1 \\ \xi_t &= \lambda_{t0} + \theta_2, \text{for } t = n_1 + 2, \dots, n_2 \end{aligned}$	(essential $ heta_1$ -equivalence) (essential $ heta_2$ -equivalence)	(107) (108)
$\begin{aligned} \xi_t &= \lambda_{t0} + \lambda_{t1} \cdot \theta_1, \text{for } t = 2, \dots, n_1 \\ \xi_t &= \lambda_{t0} + \lambda_{t1} \cdot \theta_2, \text{for } t = n_1 + 2, \dots, n_2 \end{aligned}$	$(heta_1$ -congenericity). $(heta_2$ -congenericity).	(109) (110)

Additional scale invariance assumptions (optional)

$\lambda_{t0} = \lambda_{(n_1+t)0}, \text{for } t = 2, \dots$., <i>n</i> ₁	(for ess. equivalence)	(111)
$\lambda_{t0} = \lambda_{(n_1+t)0}$ and $\lambda_{t1} =$	$\lambda_{(n_1+t)1}$, for $t = 2,, n$	(for congenericity)	(112)

Note. One of the three pairs of equivalence assumptions is made additional to the assumptions defining a multistate model (see Table 6). The variables ξ_t are defined by Equation (68).

will correlate, but this correlation does not have to be 1. (However, if the correlation is 0, then we would have to assume $\lambda_{21} = \lambda_{41} = 1$. Otherwise, the model would not be identified.)

Figure 7 represents a congeneric multistate-multitrait model omitting the timespecific latent trait variables. In this model, the two latent trait variables θ_1 and θ_2 are correlated. Figure 8 shows the same kind of model, now with the linear regression of θ_2 on θ_1 replacing the covariance between the two variables. Note that the two models are identical with respect to their implications on the expectations, variances, and covariances of the manifest variables Y_{it} .

3.7 Multigroup-Multistate-Multitrait Models

Oftentimes, we are not only interested in *describing* trait change. For example, psychotherapy should have long-term effects that are not only due to situational fluctuations. In terms of LST theory this means that an intervention should have an effect on a latent trait variable (see section 1). Multigroup-multistate-multitrait models presented in this section can be used to estimate and test average and various conditional treatment effects on a latent trait 'response' (or outcome) variable. For simplicity, we will only consider a treatment variable *X* with values 0 and 1 indicating control (*X*=0) and treatment (*X*=1). Furthermore, we restrict our presentation to the case in which we measure a latent trait variable θ_1 before treatment and a latent trait variable, θ_2 , at one appropriate time after treatment. Figure 9 depicts such a model for each of two treatment conditions. This model basically consists of a multistate-multitrait model in each of the two treatment conditions with the only exception that the *conditional mean independence assumption* now also includes



Figure 6. A congeneric multistate-multitrait model for two observations at each of two occasions of measurement. For didactic reasons, time-specific latent traits are included.



Figure 7. Two equivalent path diagrams of a congeneric multistate-multitrait model with correlated traits θ_1 and θ_2 . Time-specific latent trait variables are not represented.



Figure 8. A congeneric multistate-multitrait model with linear regression of θ_2 on θ_1

the treatment variable X, i.e., now we assume

$$E[Y_{it} | U_1, \dots, U_n, S_1, \dots, S_n, X, (Y_{is}, (j, s) \in I \times T \setminus \{(i, t)\})] = E(Y_{it} | U_t, S_t)$$
(115)

instead of Equation (64). This implies that the parameters λ_{it0} and λ_{it1} (see Table 6) of the measurement model for the latent state variables τ_t are invariant between the two treatment conditions. Invariance across all treatment conditions also holds for the parameters λ_{t0} and λ_{t1} (see Table 9) of the measurement model for the latent variables θ_t . Substantively speaking, invariance of the measurement models across treatment conditions is required if we intend to measure the same latent variables, latent states and latent traits, in both treatment conditions, and measuring the same latent variables in both treatment conditions is necessary if we want to compare their expectations and interpret their difference as an effect of the treatment.

Randomized-Experiments

In a randomized experiment, the treatment variable *X* and all pre-treatment variables will be independent. (These pre-treatment variables include the manifest variable Y_{11} , Y_{21} , Y_{12} , and Y_{22} , but also the latent state variables τ_1 and τ_2 , the latent state residuals ζ_1 and ζ_2 , as well as the latent trait variable θ_1 .) Under independence of *X* and all pre-treatment variables the *average total treatment effect* is identical to the difference

$$E(\theta_2 | X=1) - E(\theta_2 | X=0), \tag{116}$$

i.e., to the difference between the conditional expectations of θ_2 in the two treatment conditions. This difference is easily estimated and tested in a two-group structural equation model.



Figure 9. A congeneric twogroup-multistate-multitrait model.

Aside from the average total treatment effect, we can also estimate and test hypotheses about the θ_1 -*conditional-effect function*, say $g_1(\theta_1)$, that can be introduced as follows: Because *X* is dichotomous with values 0 and 1, there are always real-valued functions g_0 and g_1 such that the regression of θ_2 on *X* and θ_1 can be written:

$$E(\theta_2 | X, \theta_1) = g_0(\theta_1) + g_1(\theta_1) \cdot X.$$
(117)

Hence, for a fixed value of the pre-treatment latent trait variable θ_1 , we will always have a linear (parameterization of the) regression of *Y* on *X*. However, the intercepts and slopes of these regressions can be different for different values of θ_1 . Therefore, the values of $g_1(\theta_1)$ are the θ_1 -conditional effects of *X* on *Y*.

Now consider the treatment-specific regressions of the latent trait response variable θ_2 on the latent trait pre-treatment variable θ_1 (a latent covariate). Equation (117) immediately implies:

$$E(\theta_2 | X = 0, \theta_1) = g_0(\theta_1)$$
(118)

and

$$E(\theta_2 | X=1, \theta_1) = g_0(\theta_1) + g_1(\theta_1).$$
(119)

Hence, the effect function is

$$g_1(\theta_1) = E(\theta_2 | X=1, \theta_1) - E(\theta_2 | X=0, \theta_1),$$
(120)

the difference between the two treatment-specific regressions of θ_2 on θ_1 . If we assume that these two regressions have a linear parameterization, i.e., if they can be

written

$$E(\theta_2 | X=0, \theta_1) = \beta_0^{(0)} + \beta_1^{(0)} \cdot \theta_1 \quad \text{and} \quad E(\theta_2 | X=0, \theta_1) = \beta_0^{(1)} + \beta_1^{(1)} \cdot \theta_1, \quad (121)$$

then the effect function can easily be estimated and tested in a two-group structural equation model, because, in this case,

$$g_{1}(\theta_{1}) = E(\theta_{2} | X=1, \theta_{1}) - E(\theta_{2} | X=0, \theta_{1})$$

$$= \beta_{0}^{(1)} + \beta_{1}^{(1)} \cdot \theta_{1} - (\beta_{0}^{(0)} + \beta_{1}^{(0)} \cdot \theta_{1})$$

$$= (\beta_{0}^{(1)} - \beta_{0}^{(0)}) + (\beta_{1}^{(1)} - \beta_{1}^{(0)}) \cdot \theta_{1}$$

$$= \gamma_{10} + \gamma_{11} \cdot \theta_{1}.$$
(122)

Estimating this effect function is of substantive interest whenever we would like to know how the treatment effect depends on the pre-treatment latent trait. How big is the treatment effect for persons with a high value and how big is it for persons with a low value on θ_1 , the pre-treatment trait variable?

In contrast, the average total effect [see Eq. 116] is only the expectation of these θ_1 -conditional effects, i.e., in the randomized experiment,

$$E[g_1(\theta_1)] = E(\theta_2 | X=1) - E(\theta_2 | X=0),$$
(123)

where

$$E[g_1(\theta_1)] = E(\gamma_{10} + \gamma_{11} \cdot \theta_1) = \gamma_{10} + \gamma_{11} \cdot E(\theta_1).$$
(124)

Quasi-Experiments

While the answer to these questions is straight-forward in the framework of a randomized experiment, it is more complicated and needs more assumptions in a nonequivalent control group design. In order to identify average and conditional total treatment effects, we have to assume *unbiasedness* of the regressions $E^{X=x}(\theta_2 | \theta_1)$ in both treatment groups (see, ?, ?, for a definition as well as necessary and sufficient conditions of unbiasedness). In informal terms, unbiasedness means that there are no omitted confounding variables that introduce bias. In a randomized experiment, unbiasedness of the regressions $E(\theta_2 | X=x, \theta_1)$ always holds, whereas in a quasi-experiment, it does not necessarily hold.

However, under unbiasedness, the effect function $g_1(\theta_1)$ still informs about the θ_1 -conditional total treatment effects, and its expectation [see Eq. (124)] is still identical to the average total treatment effect. Furthermore, we can also consider

$$E[g_1(\theta_1) | X=x] = E(\gamma_{10} + \gamma_{11} \cdot \theta_1 | X=x) = \gamma_{10} + \gamma_{11} \cdot E(\theta_1 | X=x),$$
(125)

the (X=x)-conditional total treatment effect. In the non-equivalent control group design, $E[g_1(\theta_1) | X=0]$ and $E[g_1(\theta_1) | X=1]$ will differ. This means that the *average total treatment effect on the treated*, $E[g_1(\theta_1) | X=1]$, will differ from the *average total treatment effect on the nontreated*, $E[g_1(\theta_1) | X=0]$. Whereas $E[g_1(\theta_1) | X=1]$ informs about the average total treatment effect in the population of those subjects that, under the current assignment regime, choose or are selected into treatment, $E[g_1(\theta_1) | X=0]$ informs about the average total treatment effect in the population of those subjects that, under the current assignment regime, do not choose or are not selected into treatment.

4 DISCUSSION

If the treatment-specific regressions $E(\theta_2|X=x, \theta_1)$ are *not* unbiased, one can select additional covariates Z_1, \ldots, Z_q such that unbiasedness holds for the regressions $E(\theta_2 | X=x, \theta_1, Z)$, where $Z = (Z_1, \ldots, Z_q)$. In this case, the (θ_1, Z) -conditional effect function is

$$g_1(\theta_1, Z) = E(\theta_2 | X=1, \theta_1, Z) - E(\theta_2 | X=0, \theta_1, Z),$$
(126)

and the average total treatment effect is $E[g_1(\theta_1, Z)]$. Again, we can consider the (X=x)-conditional total treatment effects $E[g_1(\theta_1, Z) | X=x]$, x = 0, 1. Furthermore, we may then be interested in (X=x)-conditional total-treatment-effect functions such as $E[g_1(\theta_1, Z) | X=x, \theta_1]$ or $E[g_1(\theta_1, Z) | X=x, Z_1]$ informing how the (X=x)-conditional total treatment effects depend on θ_1 or Z_1 , respectively.

4 Discussion

In this paper we presented a revision of LST theory, a probabilistic theory of latent states and traits that deals with two fundamental problems of psychological research. Observations are *fallible* and they are *never made in a situational vacuum*. While the first problem necessitates considering measurement error, the latter requires allowing for situational fluctuations. Compared to the previous version of LST theory, our revision differs in the concept of a person, and this new concept of a person has implications on the concepts of latent traits and latent states.

4.1 What is a person?

The concept of a person has been neglected in methodology for a long time. In almost all statistical models, a person is just an index. Even, in the previous version of LST theory, a person was just a value of the person variable U. Although this lead to using more sophisticated concepts such as the conditional expectations $E(Y_{it}|U)$ and $E(Y_{it}|U, S_t)$, this static concept of a person did not allow for an adequate representation of the stochastic processes inevitably involved in longitudinal studies. In the revised version of LST theory not only traits but also persons themselves can change over time. A concrete *person-at-time-t*, denoted u_t , is a combination of the initial person u_0 and the complete past of the process, which consists of the past experiences, past situations in which the person has been assessed, and the observations made in these assessments. It differs from a concrete *person-at-time-t-ina-situation-s_t* by the concrete situation s_t in which the person is assessed at time t.

4.2 What is a trait?

If we consider manifest numerical random variables (observables) in the kind of random experiments described in section 3.1, then each such observable Y_{it} can be decomposed into the sum of a latent state τ_{it} and a measurement error component ε_{it} , and the latent state τ_{it} itself is a latent trait ξ_{it} plus a latent state residual ζ_{it} . It should be emphasized that the only prerequisite of this decomposition is that the observable Y_{it} is nonnegative or has a finite expectation. Hence, the distinction between latent states, traits, and state residuals is not based on any model, e.g., as considered in structural equation modeling. Nevertheless, the definitions of these concepts already imply a number of properties some of which are essential for understanding their meaning.

4 DISCUSSION

Perhaps, the most obvious implication is that the trait score $E(Y_{it} | U_t = u_t)$ of a person-at-time-*t* is actually a property of u_t , the person-at-time-*t*. Although this fact may seem trivial, it would not be clear if we would solely rely on introducing latent states and traits via a structural equation model. Using a structural equation model with latent variables alone does not define the latent variables involved, and this conceals their nature and substantive meaning.

A less obvious implication of the definition of a latent trait variable [see Eq. (32) in Table 4] is that the trait score $E(Y_{it} | U_t = u_t)$ of a person-at-time-*t* is the $(U_t = u_t)$ -conditional expectation of the partial conditional expectation $E(Y_{it} | U_t = u_t, S_t)$, i.e.,

$$E(Y_{it} | U_t = u_t) = E(\tau_{it} | U_t = u_t) = E[E(Y_{it} | U_t = u_t, S_t) | U_t = u_t].$$
(127)

In other words, a score of the latent trait variable is an expectation over the personat-time-*t*-conditional distribution of the situations in which the person-at-time-*t* might be. Hence, situations are implicitly involved in defining a latent trait. They determine the latent trait score via the person-at-time-*t*-specific distribution of the situations.

4.3 What is a state?

In this revision of LST theory we defined a latent state variable by $\tau_{it} = E(Y_{it}|U_t, S_t)$ and the latent state residual by $\zeta_{it} = \tau_{it} - \xi_{it}$. These definitions imply that a latent state variable is a sum of the latent trait variable and the latent state residual, i.e., $\tau_{it} = \xi_{it} + \zeta_{it}$, which can also be written

$$\begin{aligned} \pi_{it} &= E(Y_{it}|U_t, S_t) \\ &= E(Y_{it}|U_t) + [E(Y_{it}|U_t, S_t) - E(Y_{it}|U)] \\ &= E(Y_{it}|U_t) + E(Y_{it}|S_t) + [E(Y_{it}|U_t, S_t) - E(Y_{it}|U_t) - E(Y_{it}|S_t)] \end{aligned}$$
(128)

This equation shows that a latent state variable deviates from the latent trait variable $E(Y_{it}|U_t)$ by the latent situation variable $E(Y_{it}|S_t)$ and the person-situation interaction variable $E(Y_{it}|U_t, S_t) - E(Y_{it}|U_t) - E(Y_{it}|S_t)$.

4.4 How can we use traits in empirical research?

4.5 Implications for design of empirical studies





References

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