Individuelle, bedingte und durchschnittliche kausale Effekte.
Eine Einführung

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Example 1: The Simpson paradox

Table 1. *Evaluation of a treatment*

	treatment		
success	yes ($X = 1$)	no ($X = 0$)	total
yes ($Y = 1$)	500	600	1100
no ($Y = 0$)	500	400	900
	1000	1000	2000

Note. After Novick (1980). The numbers are fictitious.

Example 1: The Simpson paradox (2)

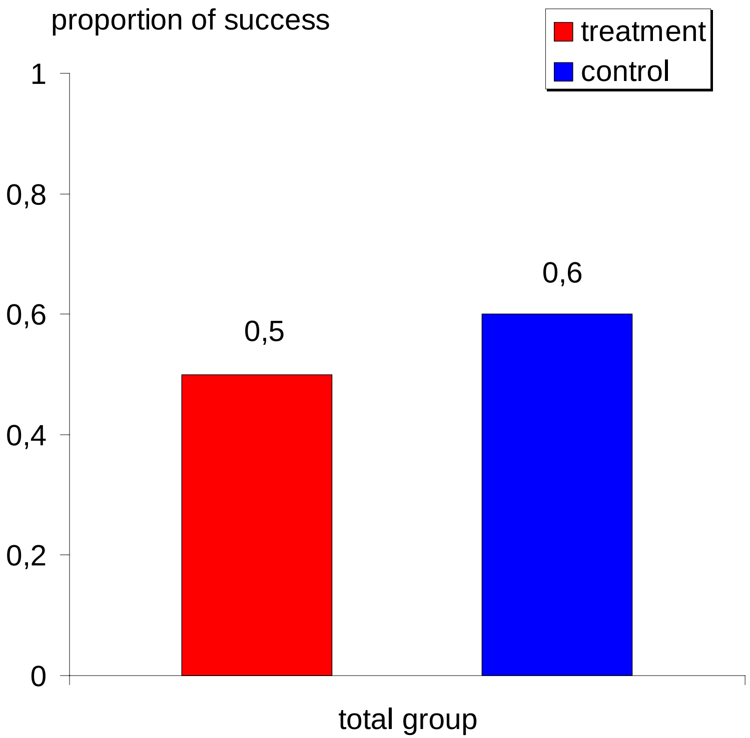


Figure 1. Histogram of the relative frequencies of success

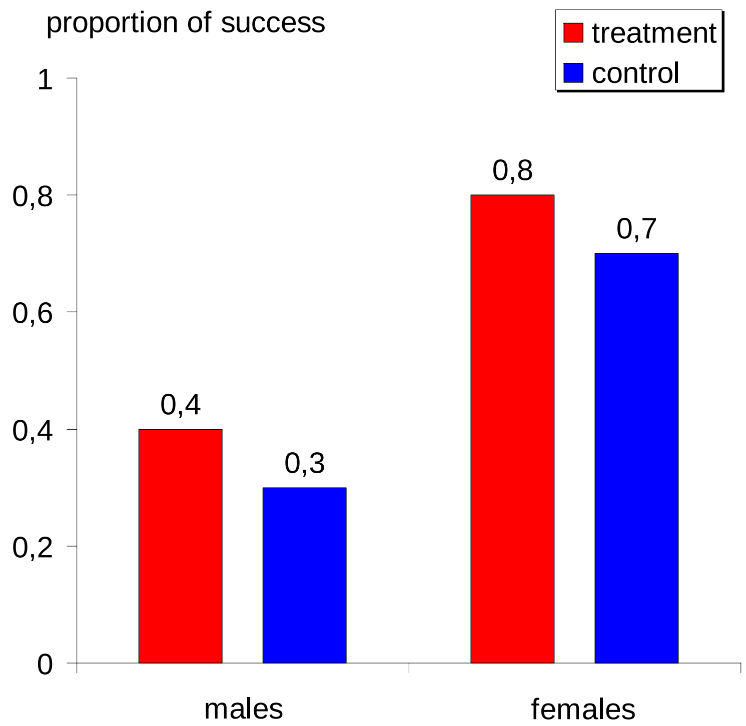


Figure 2. Histogram of the relative frequencies for success within gender groups

Example 1: The Simpson paradox (3)

Table 2. *Evaluation of a treatment within gender groups*

A. Males ($W = 1$)

	treatment		
success	yes ($X = 1$)	no ($X = 0$)	total
yes ($Y = 1$)	300	75	375
no ($Y = 0$)	450	175	625
	750	250	1000

Example 1: The Simpson paradox (4)

B. Females ($W = 0$)

success	yes ($X = 1$)	no ($X = 0$)	total
yes ($Y = 1$)	200	525	725
no ($Y = 0$)	50	225	275
	250	750	1000

Note. After Novick (1980). The numbers are fictitious.

1. Theorie

Individuelle, bedingte und durchschnittliche kausale Effekte (Neyman, Rubin)

Person	$P(U=u)$ Sampling probability	$\tau_0(u) = E(Y \mid X = 0, U = u)$ Expected outcome under control	$\tau_1(u) = E(Y \mid X = 1, U = u)$ Expected outcome under treatment	$\tau_{1\text{-}0}(u) = E(Y \mid X = 1, U = u) - E(Y \mid X = 0, U = u)$ Individual causal effect
u_1	1/8	68	82	14
u_2	1/8	81	89	8
u_3	1/8	89	101	12
u_4	1/8	102	108	6
u_5	1/8	112	118	6
u_6	1/8	119	131	12
u_7	1/8	131	139	8
u_8	1/8	138	152	14

$E[\tau_0(U)] = 105$ 115 $= E[\tau_1(U)]$

$ACE := E[\tau_{1\text{-}0}(U)] = E[\tau_1(U)] - E[\tau_0(U)] = 10$

Define $\tau_0(U)$, $\tau_1(U)$ and $\tau_{1\text{-}0}(U)$ by:

$$\tau_0(u) := E(Y \mid X = 0, U = u)$$

$$\tau_1(u) := E(Y \mid X = 1, U = u)$$

$$\tau_{1\text{-}0}(u) = \tau_1(u) - \tau_0(u)$$

$\tau_{1\text{-}0}(u)$: *individual causal effect of unit u*

$E[\tau_{1\text{-}0}(U)] = E[\tau_1(U)] - E[\tau_0(U)]$

= *average causal effect ACE*

$E[\tau_{1\text{-}0}(U) \mid Z=z]$ =:*conditional causal effect*

$E[\tau_{1\text{-}0}(U) \mid X=x]$ = *average causal effect of the treated*

If X is dichotomous with values 0 and 1, then:

$$E(Y \mid X, U) = \tau_0(U) + \tau_{1\text{-}0}(U) \cdot X$$

Verfälschung von Mittelwertsdifferenzen (prima facie Effekten)

$$E(Y|X{=}x) = \sum_u E(Y|X{=}x, U{=}u)\, P(U{=}u\,|X{=}x)$$

$$E(\tau_x) = \sum_u E(Y|X{=}x, U{=}u)\, P(U{=}u)$$

Table 4.1: An example in which the prima facie effect is biased

Unit	Sampling probability $P(U=u)$	Expected outcome under treatment $E(Y \mid X=0, U=u)$	Expected outcome under control $E(Y \mid X=1, U=u)$	Individual causal effect $E(Y \mid X=1, U=u) - E(Y \mid X=0, U=u)$	Probability for control $P(X=0 \mid U=u)$	Probability for treatment $P(X=1 \mid U=u)$
u_1	1/8	68	82	14	1/9	8/9
u_2	1/8	78	86	8	2/9	7/9
u_3	1/8	88	100	12	3/9	6/9
u_4	1/8	98	104	6	4/9	5/9
u_5	1/8	108	114	6	5/9	4/9
u_6	1/8	118	130	12	6/9	3/9
u_7	1/8	128	136	8	7/9	2/9
u_8	1/8	138	152	14	8/9	1/9
				Average causal effect		10.00
				Prima facie effect		−13.33

Note. All numbers displayed for observational units are fictitious.

1. Theorie: Generalisierung für *J* + 1 Treatment-Bedingungen

$$E(Y \mid X, U) = \tau_0(U) + \tau_{1\text{-}0}(U) \cdot I_{X=1} + \ldots + \tau_{J\text{-}0}(U) \cdot I_{X=J}$$

where $I_{X=j} = \begin{cases} 1, \text{ if } X = j \\ 0, \text{otherwise} \end{cases}$

We then have J ICE-functions $\tau_{j\text{-}0}(U)$
containing the effects of treatment j
compared to the reference treatment 0 (control).

3. Bedingte und durchschnittliche (prima facie) Effekte

$$E(Y \mid X, Z) = g_0(Z) + g_{1\text{-}0}(Z) \cdot X$$

where $g_{1\text{-}0}(z)$ is the conditional causal effect given $Z = z$ and $E[\, g_1(Z)]$ is the average effect **if certain assumptions hold**.

3. Bedingte und durchschnittliche (prima facie) Effekte

Die bedingten prima facie Effekte $g_1(z)$ sind gleich den bedingten kausalen Effekten, wenn die individuellen Treatment-Wahrscheinlichkeiten nicht von den zu erwartenden Effekten und nicht von den Expected outcomes under control abhängen.

Theorem 1 (Sufficient conditions for unbiasedness of the conditional prima facie effects)

If X is dichotomous with values 0 and 1, and

$$E(Y|X,U,Z) = E(Y|X,U) = \tau_0(U) + \tau_{1\text{-}0}(U)\cdot X, \qquad (1)$$

$$E[\ \tau_0(U)\,|\,X,Z] = E[\ \tau_0(U)\,|\,Z], \qquad \text{weak conditional (2)}$$

$$E[\ \tau_{1\text{-}0}(U)\,|\,X,Z] = E[\ \tau_{1\text{-}0}(U)\,|\,Z], \qquad \text{ignorability (3)}$$

hold, then the values $g_{1\text{-}0}(z)$ of the effect function $g_{1\text{-}0}(Z)$ in the regression $E(Y|X, Z) = g_0(Z) + g_{1\text{-}0}(Z)\cdot X$, are the conditional causal effects of X on Y, i.e.:

$$g_{1\text{-}0}(Z) = E[\ \tau_{1\text{-}0}(U)\,|\,Z] = CCE(Z), \qquad \text{unbiasedness (4)}$$

and the average causal effect can be computed by

$$ACE = E[g_{1\text{-}0}(Z)]. \qquad \text{adjustment formula (5)}$$

3. Bedingte und durchschnittliche (prima facie) Effekte

$J + 1$ treatment conditions:

$$E(Y \mid X, Z) = g_0(Z) + g_{1\text{-}0}(Z) \cdot I_{X=1} + \ldots + g_{J\text{-}0}(Z) \cdot I_{X=J}$$

3. Standard Forschungsfragen

Suppose we just have 2 treatment conditions

$$E(Y \mid X, Z) \;=\; g_0(Z) + g_{1\text{-}0}(Z) \cdot I_{X=1}$$

Then the standard research questions are:

- What are the conditional effects of treatment as compared to the control given Z?
  Hence, we want to
  (a) **estimate the effect function** $g_{1\text{-}0}(Z)$ and
  (b) test H_0: $g_{1\text{-}0}(Z) = \gamma_{00}$ (constant) **no interaction**

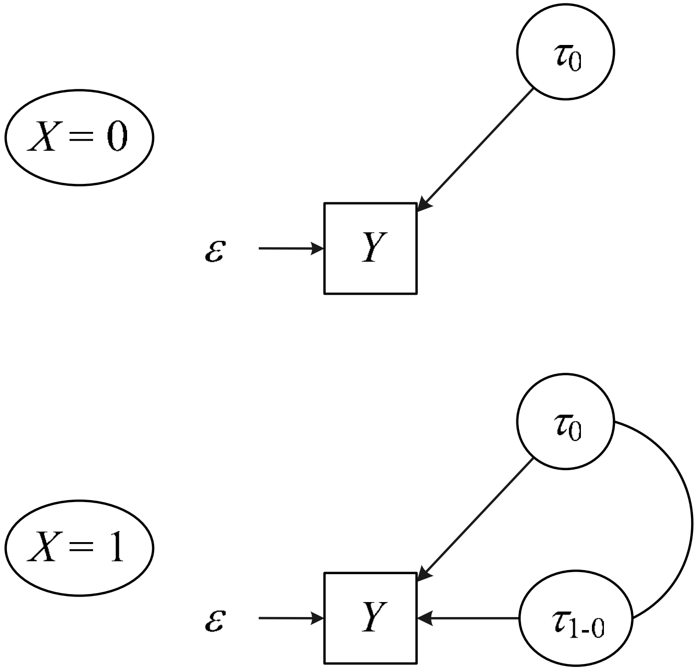
- What is the average effect of treatment as compared to the control?
  Hence, we want
  (c) to **estimate the average effect** $E[g_{1\text{-}0}(Z)]$ and
  (d) test H_0: $E[g_{1\text{-}0}(Z)] = 0$ **no average effect**

Not yet identified ICE model

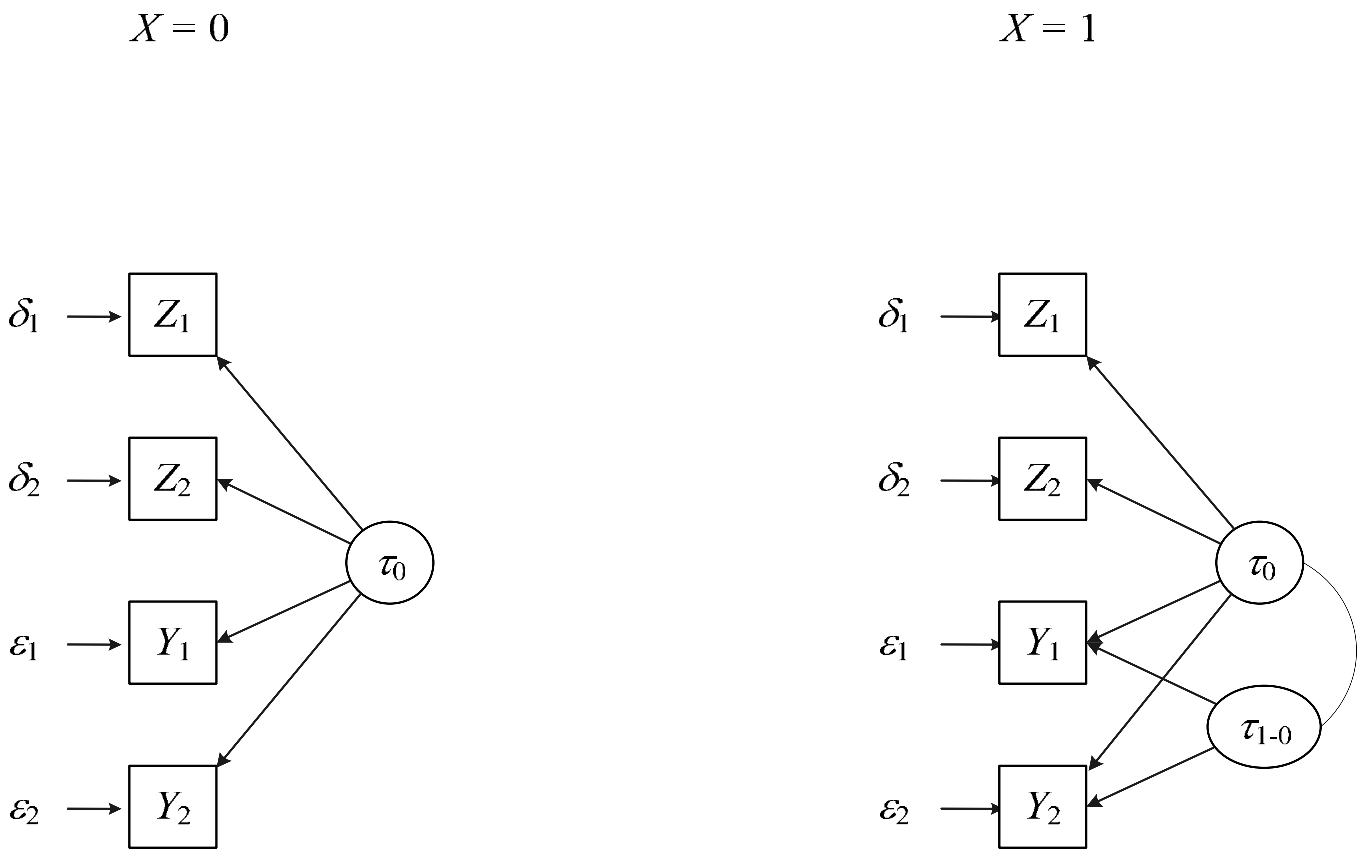
$$E(Y \mid X, U) = \tau_0(U) + \tau_{1\text{-}0}(U) \cdot X \quad \text{or} \quad Y = \tau_0(U) + \tau_{1\text{-}0}(U) \cdot X + \varepsilon$$

$$Var_{X=1}(Y) = Var_{X=1}(\tau_0 + \tau_{1\text{-}0} + \varepsilon)$$

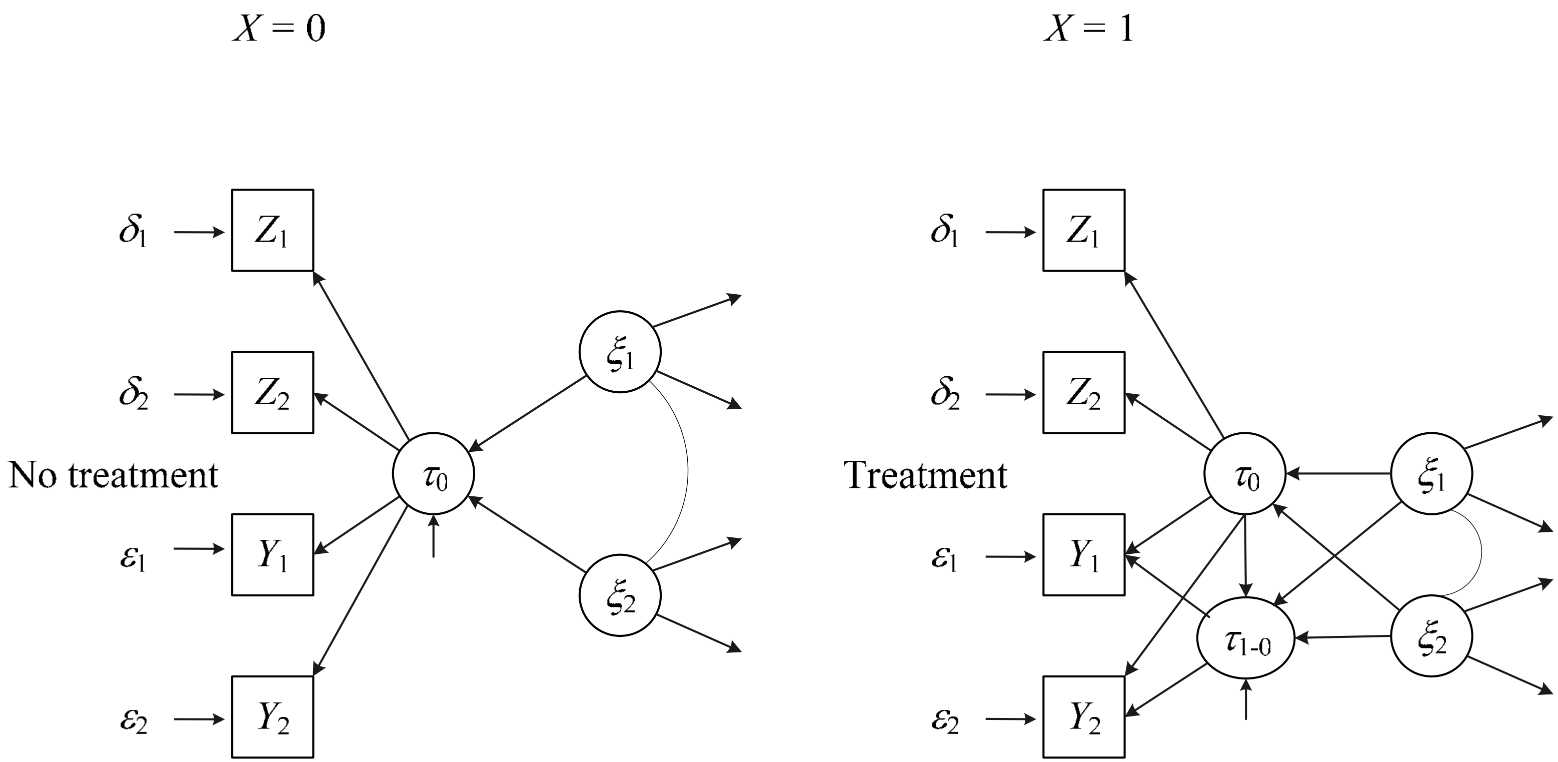
$$= Var_{X=1}(\tau_0) + Var_{X=1}(\tau_{1\text{-}0}) + 2\,Cov_{X=1}(\tau_0, \tau_{1\text{-}0}) + Var_{X=1}(\varepsilon)$$



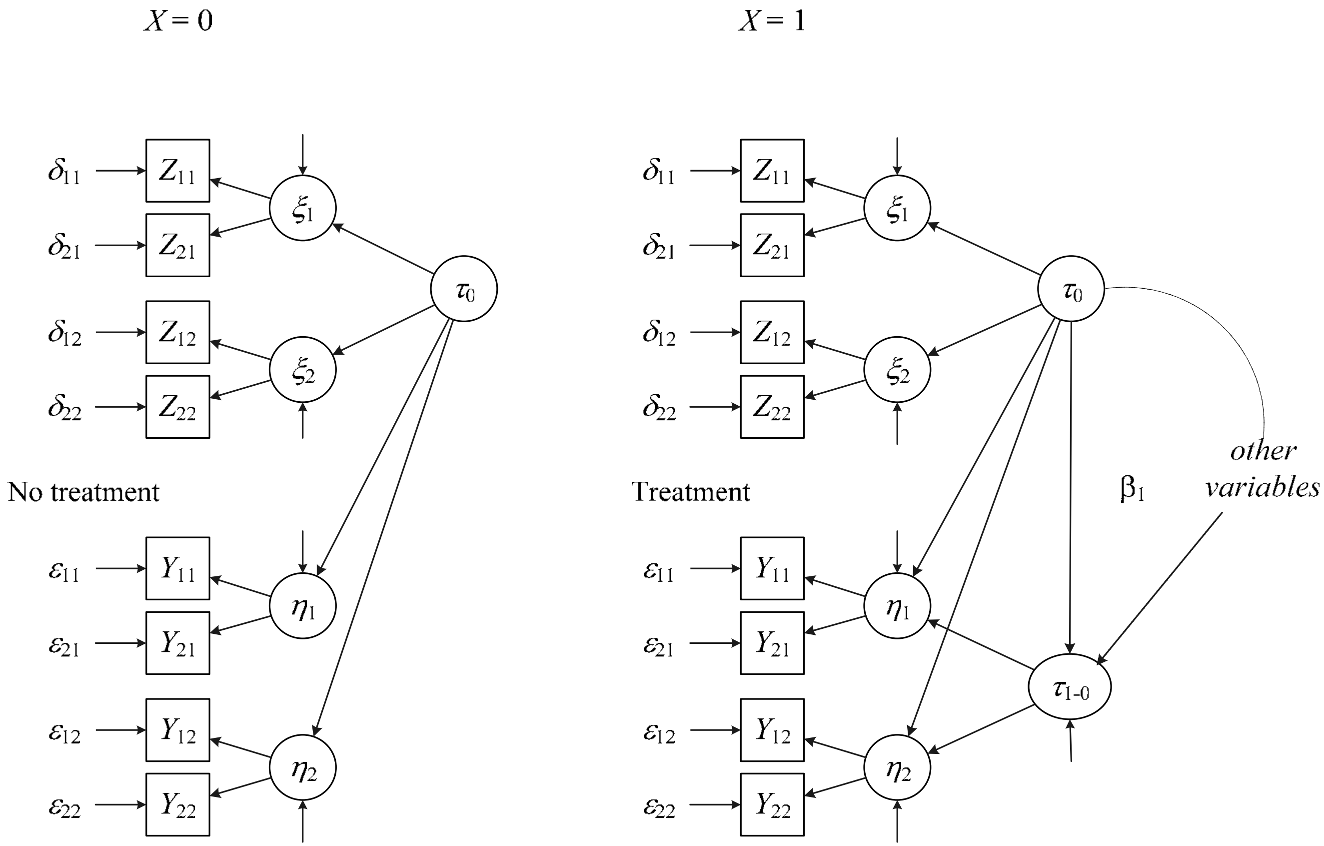
Identified ICE Model with pretests Z_1 and Z_2



Explaining individual causal effects by other (latent) variables



LST Individual Effects Model



Steyer, R. (2005). Analyzing Individual and Average Causal Effects via Structural Equation Models. *Methodology: European Journal of Research Methods in the Behavioral and Social Sciences*, *1*, 39-54.